MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-GEO-CIRCLES
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · SHAPE

CIRCLE THEOREMS

Seven theorems, and a reason for every angle. Secure spotting each theorem, extend to tangents and isosceles triangles hiding in radii, then master multi-step chains and proof.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

The angle at the **centre** is twice the angle at the circumference. The angle in a **semicircle** is 90°. Angles in the **same segment** are equal. Opposite angles of a **cyclic quadrilateral** add to 180°.

A **tangent** meets a radius at 90°. Two tangents from one point are **equal**. The angle between a tangent and a chord equals the angle in the **alternate segment**. The perpendicular from the centre to a chord **bisects** it. Two radii make an **isosceles** triangle.

1. Mark every radius: two radii mean an isosceles triangle.
2. Look for the shape the theorem needs: a 'bow tie', an 'arrowhead', a quadrilateral, a tangent.
3. Write the angle and the theorem's reason, every time. Reasons earn marks.
4. Use ordinary angle facts too: triangles 180°, straight lines 180°.

WORKED EXAMPLE **A, B and C are on a circle with centre O. Angle OAB = 20°. Find angle ACB, giving reasons.**

SOLUTION

1 OA = OB (radii), so triangle OAB is isosceles: angle OBA = 20°.
2 Angle AOB = 180° − 20° − 20° = 140° (angles in a triangle add to 180°).
3 The angle at the centre is twice the angle at the circumference.
4 So angle ACB = 140° ÷ 2 = 70°.

✓ **Angle ACB = 70°**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
One theorem at a time, with the reason written.
Secured ☐

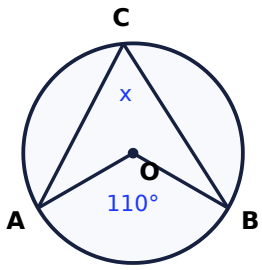
Extending
Tangents from a point, radii, the alternate segment, algebra, and tampered evidence.
Secured ☐

Mastery
Chains of theorems, proofs, and chords.
Secured ☐

SECURING

The method, step by step. Show every line.

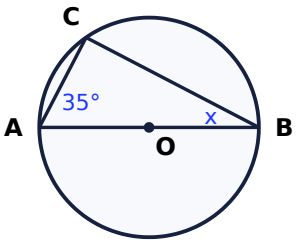
1 O is the centre of the circle. Find x, giving a reason. [2]



Not drawn accurately

x = _____

2 AB is a diameter. Find x, giving reasons. [2]



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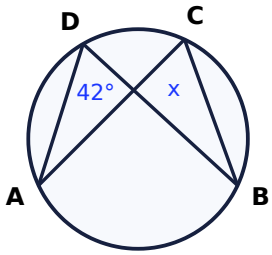
x = _____

3 Find x, giving a reason. [2]



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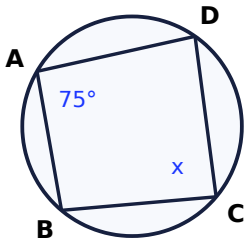
x = _____

FORENSIC TIP Look for the 'bow tie': two angles standing on the same chord AB, on the same side.

4 ABCD is a cyclic quadrilateral. Find x, giving a reason. [2]



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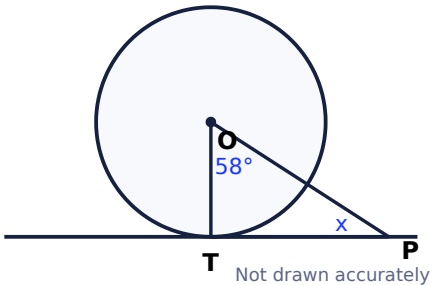
x = _____

5 PT is a tangent to the circle at T, and O is the centre. Find x, giving reasons. [2]



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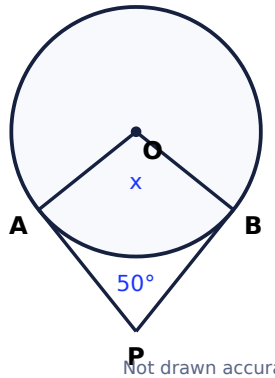
x = _____

EXTENDING A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 PA and PB are tangents to the circle with centre O. Find x, giving reasons. [3]



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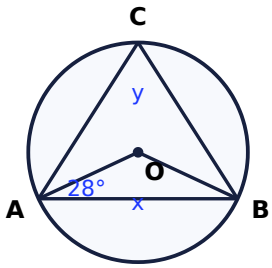
x = _____

7 O is the centre. Find x and y, giving reasons. [3]



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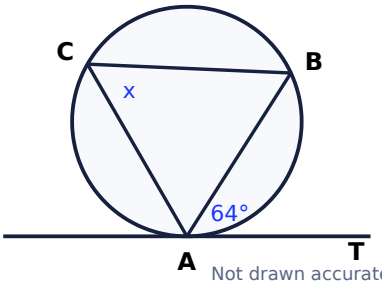
x = ___ y = _____

FORENSIC TIP Two radii make an isosceles triangle. This is the step most students miss.

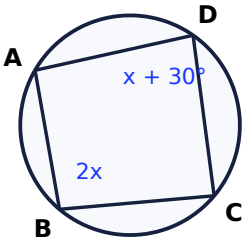
8 The line through A is a tangent to the circle. Find x, giving a reason. [2]



Not drawn accurately

x = _____

9 ABCD is a cyclic quadrilateral. Find x. [3]



Not drawn accurately

x = _____

10 A point C is on a circle with centre O. Angle AOB = 140°. Kim wrote the line below. Find her mistake. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

angle ACB = 2 × 140° = 280°

angle ACB = _____

MASTERY No hints. Plan before you calculate.

11 AB is a diameter of a circle with centre O. C is on the circle. The tangent at C meets AB extended at P, and angle CPA = 30°. [4]

Find angle CAP, giving reasons.

angle CAP = _____

12 Use the angle at the centre theorem to prove that opposite angles of a cyclic quadrilateral add up to 180°. [4]

13 In a cyclic quadrilateral ABCD, angle A = $2y$, angle B = $3x$, angle C = $4y - 30°$ and angle D = $x + 40°$. Find x and y. [4]

x = ___ y = _____

14 PA and PB are tangents to a circle with centre O, and angle APB = 40°. C is a point on the major arc AB. Find angle ACB, giving reasons. [3]

angle ACB = _____

15 A circle has radius 13 cm. A chord of the circle is 24 cm long. Find the distance from the centre to the chord. [3]

cm _____

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

The angle at the centre is twice the angle at the circumference.

✓ **55°**

Q2 SECURING

The angle in a semicircle is 90°, so x = 180° − 90° − 35°.

✓ **55°**

Q3 SECURING

Angles in the same segment are equal.

✓ **42°**

Q4 SECURING

Opposite angles of a cyclic quadrilateral add to 180°.

✓ **105°**

Q5 SECURING

A tangent meets a radius at 90°, so x = 180° − 90° − 58°.

✓ **32°**

Q6 EXTENDING

Angles OAP and OBP are 90° (tangent meets radius). Angles in a quadrilateral: 360° − 90° − 90° − 50°.

✓ **130°**

Q7 EXTENDING

OA = OB (radii) so angle OBA = 28°; x = 180° − 56°. y = x ÷ 2 (angle at the centre).

✓ **x = 124°, y = 62°**

Q8 EXTENDING

Alternate segment theorem.

✓ **64°**

Q9 EXTENDING

2x + x + 30° = 180°.

✓ **x = 50°**

Q10 EXTENDING

The angle at the centre is **twice** the angle at the circumference, so halve it.

✓ **70°**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-GEO-CIRCLES

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	10		☐
Extending	13		☐
Mastery	18		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: CIRCLE THEOREMS

In this case file you've learned to:

- Recognise all seven circle theorems in a diagram.
- Write a reason for every angle you find.
- Spot the isosceles triangle made by two radii.
- Combine theorems with angle facts in multi-step chains.
- Prove a circle theorem from another.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab