MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-GEO-CONGRUENCE
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · SHAPE

CONGRUENCE

Same shape, same size, proved beyond doubt. Secure the four conditions, extend to proofs in quadrilaterals, then master proofs that use circles and isosceles triangles.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

Two triangles are **congruent** if they are exactly the same shape and size. You only need to show one of four conditions: **SSS** (three sides), **SAS** (two sides and the angle between them), **ASA** (two angles and a corresponding side) or **RHS** (right angle, hypotenuse, side).

AAA is **not** enough: it proves similar, not congruent. A congruence proof is a short list of three facts, each with a reason, then the condition.

1. Name the two triangles, with the letters in matching order.
2. Find three pairs of equal sides or angles.
3. Give a reason for each: 'opposite sides of a parallelogram', 'common side', 'radii'.
4. State the condition (SSS, SAS, ASA or RHS), then what follows from it.

WORKED EXAMPLE **ABCD is a parallelogram. Prove that triangles ABC and CDA are congruent.**

SOLUTION

1 AB = CD (opposite sides of a parallelogram are equal).
2 BC = DA (opposite sides of a parallelogram are equal).
3 AC = CA (common side).
4 So triangles ABC and CDA are congruent (SSS).

✓ **Congruent by SSS**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
SSS, SAS, ASA and RHS, and picking the right one.
Secured ☐

Extending
Proofs inside parallelograms, kites and isosceles triangles, and tampered evidence.
Secured ☐

Mastery
Multi-step proofs, and what congruence lets you conclude.
Secured ☐

SECURING

The method, step by step. Show every line.

1 Write down the four conditions for two triangles to be congruent. [2]

2 Triangle P has sides 5 cm and 7 cm with an angle of 40° between them. Triangle Q has the same. Which condition shows they are congruent? [1]

Answer _____

3 Two triangles both have angles of 50°, 60° and 70°. Must they be congruent? Explain. [2]

Yes / No _____

4 In triangles ABC and DEF, AB = DE, BC = EF and AC = DF. State the condition, and name the angle equal to angle A. [2]

___ ; angle _____

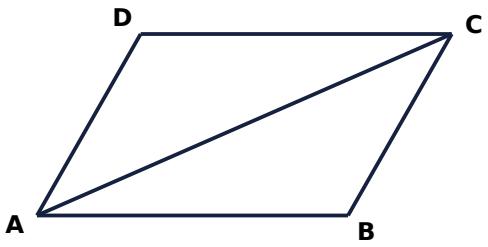
5 Two right-angled triangles each have a hypotenuse of 10 cm and another side of 6 cm. Which condition shows they are congruent? [1]

Answer _____

EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 ABCD is a parallelogram. Prove that angle ABC = angle CDA. [3]



Not drawn accurately

FORENSIC TIP Once triangles are congruent, every matching side and angle is equal. That's how a proof gets you to the fact you want.

7 Triangle ABC has AB = AC. M is the midpoint of BC. Prove that triangles ABM and ACM are congruent, and explain why AM is perpendicular to BC. [3]

8 ABCD is a kite with AB = AD and CB = CD. Prove that triangles ABC and ADC are congruent. [3]

9 ABCD is a square. E is on AB and F is on BC, with AE = BF. Prove that triangles DAE and ABF are congruent. [3]

10 Sam's proof is below. Find the flaw. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

Both triangles have angles 30°, 60°, 90° and a side of 5 cm.
So they are congruent (ASA).

MASTERY

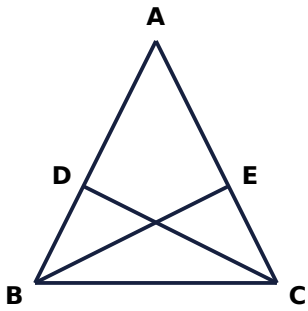
No hints. Plan before you calculate.

11 PQRS is a parallelogram. X is on PQ and Y is on RS, with PX = RY. Prove that triangles PXS and RYQ are congruent. [4]

12 Prove that the diagonals of a parallelogram ABCD bisect each other. Let them meet at M. [4]

13 A and B lie on a circle with centre O. M is the midpoint of the chord AB. Prove that OM is perpendicular to AB. [4]

14 ABC is isosceles with AB = AC. D is on AB and E is on AC, with BD = CE. Prove that BE = CD. [4]



Not drawn accurately

FORENSIC TIP Choose the triangles that contain the two lengths you need: BE is in triangle EBC, CD is in triangle DCB.

15 Explain why 'two sides and an angle that is not between them' (SSA) is not a condition for congruence. [2]

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

✓ SSS, SAS, ASA, RHS

Q2 SECURING

✓ SAS

Q3 SECURING

✓ No: equal angles show they're similar, but one could be bigger.

Q4 SECURING

✓ SSS; angle D

Q5 SECURING

✓ RHS

Q6 EXTENDING

✓ AB = CD and BC = DA (opposite sides of a parallelogram), AC common, so ABC ≅ CDA (SSS). Corresponding angles ABC and CDA are equal.

Q7 EXTENDING

✓ AB = AC (given), BM = CM (M is the midpoint), AM common: SSS. So angles AMB and AMC are equal, and they add to 180°, so each is 90°.

Q8 EXTENDING

✓ AB = AD, CB = CD (given), AC common: SSS.

Q9 EXTENDING

✓ DA = AB (sides of a square), AE = BF (given), angle DAE = angle ABF = 90°: SAS.

Q10 EXTENDING

✓ The 5 cm sides don't correspond: one is opposite 30°, the other opposite 60°. ASA needs a corresponding side.

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-GEO-CONGRUENCE

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	8		☐
Extending	14		☐
Mastery	18		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: CONGRUENCE

In this case file you've learned to:

- Use SSS, SAS, ASA and RHS, and know AAA isn't enough.
- Match corresponding vertices when naming triangles.
- Give a reason for every fact in a proof.
- Use congruence to prove lengths and angles equal.
- Prove properties of parallelograms, kites, isosceles triangles and circles.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab