MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-GEO-EXACTTRIG
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · SHAPE

# EXACT TRIG VALUES

No calculator, no rounding. Secure the values for 0°, 30°, 45°, 60° and 90°, extend to finding sides and angles exactly, then master surd answers in real shapes.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

## CASE BRIEFING

**WHAT YOU NEED TO KNOW**

On the non-calculator paper you must know these values by heart. They come from two triangles: half an equilateral triangle (30° and 60°) and half a square (45°).

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| **sin** | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| **cos** | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| **tan** | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | – |

1. Learn the table, or rebuild it from the two triangles.
2. Label the sides of the right-angled triangle: hypotenuse, opposite, adjacent.
3. Pick sin, cos or tan with SOH CAH TOA, and substitute the exact value.
4. Keep surds exact: rationalise a denominator if you're asked to simplify.

**WORKED EXAMPLE** **A ramp makes an angle of 30° with the ground and is 8 m long. How high does it rise? Give an exact answer.**

**SOLUTION**

1 The ramp is the hypotenuse; the height is opposite the 30° angle. So use sin.

2 $\sin 30° = \frac{\text{height}}{8}$.

3 $\sin 30° = \frac{1}{2}$, so height = $8 \times \frac{1}{2}$.

4 Height = 4 m. Check: 4 is half of 8, as the 30° triangle always gives ✓

**✓ 4 m**

## KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

**Securing**
Recall the exact values and combine them.
Secured ☐

**Extending**
Exact sides and angles in right-angled triangles, and tampered evidence.
Secured ☐

**Mastery**
Surd answers in shapes, and simple trig equations.
Secured ☐

### SECURING

The method, step by step. Show every line.

**1** Write down the exact value of sin 30°. [1]

Answer _____

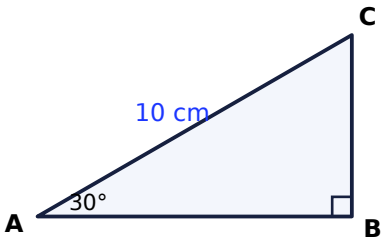
**2** Work out cos 60° + sin 30°. [2]

Answer _____

**3** Work out tan 45° × cos 0°. [2]

Answer _____

**4** Find the exact length of BC. [2]




Not drawn accurately

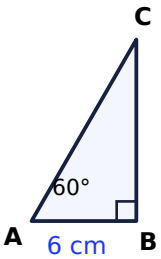
cm _____

**5** Write down the exact value of tan 60°. [1]

Answer _____

**EXTENDING** A step is hidden, or there's a context to unpick. One piece of tampered evidence.

**6** Find the exact length of the hypotenuse AC. [2]




Not drawn accurately

cm _____

**7** Work out the exact value of sin 60° × tan 30°. [2]

Answer _____

**8** A right-angled triangle has an opposite side of 4 cm and a hypotenuse of 8 cm. Find the angle without a calculator. [2]

° _____

**9** Show that $\sin^2 45° + \cos^2 45° = 1$. [2]

**10** Ola wrote this. Find her mistake. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$\sin 30° + \sin 30° = \sin 60°$

Correct value _____

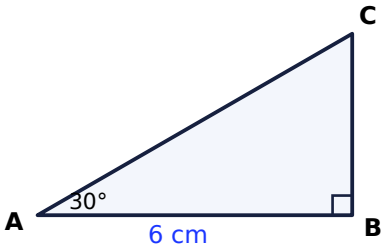
**MASTERY** No hints. Plan before you calculate.

**11** An equilateral triangle has sides of 6 cm. Find its exact area. [3]

cm² _____

**FORENSIC TIP** Split it into two right-angled triangles with angles 30°, 60°, 90°. The height is the side opposite 60°.

**12** Find the exact length of BC. Give your answer in its simplest form. [3]



Not drawn accurately

cm _____

**13** Solve $2 \sin x = 1$ for $0° \leq x \leq 90°$. [2]

$x$ = _____

**14** Show that $\cos 30° \times \tan 60° = 3/2$. [2]

**15** Triangle ABC has a right angle at C. AB = 8 cm and angle A = 45°. Find the exact length of AC. [3]

cm _____

## KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

**Q1** SECURING

✓ ½

**Q2** SECURING

½ + ½.

✓ **1**

**Q3** SECURING

1 × 1.

✓ **1**

**Q4** SECURING

sin 30° × 10.

✓ **5 cm**

**Q5** SECURING

✓ **√3**

**Q6** EXTENDING

cos 60° = 6 ÷ AC, so AC = 6 ÷ ½.

✓ **12 cm**

**Q7** EXTENDING

$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$ = ½.

✓ **½**

**Q8** EXTENDING

sin θ = ½.

✓ **30°**

**Q9** EXTENDING

✓ **(√2/2)² + (√2/2)² = ½ + ½ = 1**

**Q10** EXTENDING

✓ **sin 30° + sin 30° = ½ + ½ = 1, but sin 60° = √3/2. You can't add the angles.**

**Mastery solutions are in the Lab.** Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-GEO-EXACTTRIG

### Mark your evidence

| Level | Marks | My score | Secured? (every mark) |
|---|---|---|---|
| Securing | 8 | | ☐ |
| Extending | 10 | | ☐ |
| Mastery | 13 | | ☐ |

**R · Reading**
Misread or missed part of the question

**M · Method**
Didn't know how to do it

**A · Accuracy**
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

## CASE CLOSED: EXACT TRIG VALUES

In this case file you've learned to:

- Recall sin, cos and tan of 0°, 30°, 45°, 60° and 90°.
- Rebuild the values from half an equilateral triangle and half a square.
- Find sides and angles exactly in right-angled triangles.
- Give answers in simplest surd form.
- Solve simple trig equations without a calculator.

**Because every case comes down to one principle: numbers don't lie.**


YOUR NEXT ASSIGNMENT

### Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab