MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-PROB-TREE
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · DATA AND PROBABILITY

PROBABILITY

Every outcome, every branch. Secure single events and expected results, extend to tree diagrams, then master without-replacement problems and algebra.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

Probabilities add up to 1. P(not A) = 1 − P(A). For **independent** events, multiply: P(A and B) = P(A) × P(B). For outcomes that can't happen together, add: P(A or B) = P(A) + P(B).

On a **tree diagram**, multiply **along** the branches and add **between** the routes you want. **Without replacement**, the second set of branches changes: one fewer item, and one fewer of the colour you took.

1. Write the probabilities on every branch. Each set of branches adds up to 1.
2. Multiply along each route you want.
3. Add the routes together.
4. For 'at least one', it's usually quicker to do 1 − P(none).

WORKED EXAMPLE **A bag has 3 red and 5 blue counters. Two are taken without replacement. Find the probability that both are red.**

SOLUTION

1 First counter: P(red) = $\frac{3}{8}$.

2 Without replacement there are now 2 red out of 7: P(red) = $\frac{2}{7}$.

3 Multiply along the branch: $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$.

4 Simplify: $\frac{3}{28}$. Check: it's smaller than $\frac{3}{8} \times \frac{3}{8}$, as it should be ✓

✓ $\mathbf{\frac{3}{28}}$

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Single events, missing probabilities, relative frequency, listing outcomes.
Secured ☐

Extending
Tree diagrams with replacement, combined events, counting, and tampered evidence.
Secured ☐

Mastery
Without replacement, 'at least one', conditional probability and algebra.
Secured ☐

SECURING

The method, step by step. Show every line.

1 A bag has 5 red, 3 blue and 2 green counters. One is taken at random. Find P(blue). [1]

Answer _____

2 Using the same bag, find P(red or green). [2]

Answer _____

3 A spinner lands on A, B, C or D with probabilities 0.2, 0.35, x and 0.15. Find x. [2]

x = _____

4 A dice is thrown 200 times and lands on six 48 times. Estimate how many sixes there would be in 500 throws. [2]

Answer _____

5 Two fair coins are flipped. List all the possible outcomes, and find P(two heads). [2]

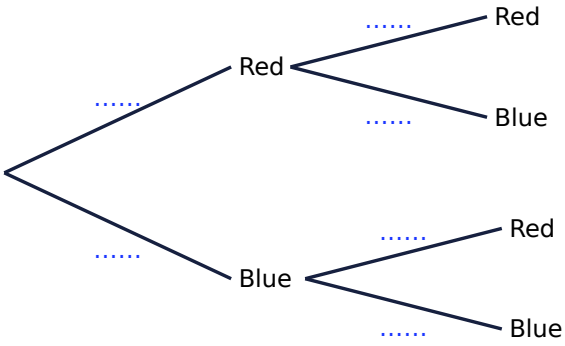
P(two heads) _____

FORENSIC TIP HT and TH are **different** outcomes. Missing one is the most common slip.

EXTENDING A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 A bag has 3 red and 2 blue counters. A counter is taken, its colour noted and it is put back. Then a second counter is taken. [3]

Complete the tree diagram, then find P(both red).

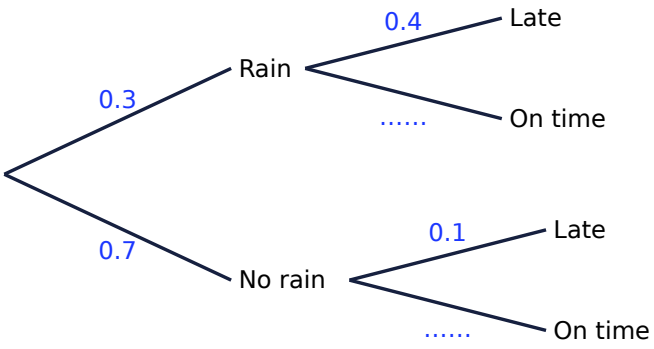


P(both red) _____

7 Using the same tree diagram, find the probability of getting one of each colour. [3]

Answer _____

8 The probability it rains on a school day is 0.3. Complete the tree diagram, then find the probability a detective trainee is late. [3]



Answer _____

9 A case code is 3 letters (A to Z) followed by 2 digits (0 to 9). How many different codes are possible? [2]

Answer _____

10 Zak found the probability of rolling two sixes with two dice. Find his mistake. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$P(6 \text{ and } 6) = 1/6 + 1/6 = 1/3$

Answer _____

MASTERY

No hints. Plan before you calculate.

11 A bag has 4 red and 6 blue sweets. Two are taken without replacement. Find the probability they are the same colour. [4]

Answer _____

12 Using the same bag, find the probability that at least one sweet is red. [3]

Answer _____

FORENSIC TIP 'At least one' covers lots of routes. Find P(none) instead and take it away from 1.

13 A bag has n sweets, 6 of them orange. Two are taken without replacement. P(both orange) = 1/3. [4]

Show that $n^2 - n - 90 = 0$, and find n.

n = _____

14 60% of a class study French. Of these, 30% also study Spanish. Of the others, 50% study Spanish. A student who studies Spanish is chosen at random. Find the probability they also study French. [4]

Answer _____

15 A fair dice is thrown three times. Find the probability of at least one six. [3]

Answer _____

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

✓ **3/10**

Q2 SECURING

5 + 2 = 7 out of 10.

✓ **7/10**

Q3 SECURING

1 − 0.7.

✓ **0.3**

Q4 SECURING

Relative frequency 48/200 = 0.24. 0.24 × 500.

✓ **120**

Q5 SECURING

✓ **HH, HT, TH, TT; P(two heads) = 1/4**

Q6 EXTENDING

3/5 × 3/5.

✓ **9/25**

Q7 EXTENDING

RB + BR = 6/25 + 6/25.

✓ **12/25**

Q8 EXTENDING

0.3 × 0.4 + 0.7 × 0.1.

✓ **0.19**

Q9 EXTENDING

26 × 26 × 26 × 10 × 10.

✓ **1 757 600**

Q10 EXTENDING

Both sixes means AND: multiply, 1/6 × 1/6.

✓ **1/36**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-PROB-TREE

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	9		☐
Extending	13		☐
Mastery	18		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: PROBABILITY

In this case file you've learned to:

- Use the fact that probabilities add up to 1.
- Estimate with relative frequency and expected outcomes.
- Draw and use tree diagrams: multiply along, add between.
- Handle without-replacement problems by changing the second branches.
- Use 1 − P(none), conditional probability and algebra.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab