MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-GEO-PYTHTRIG
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · SHAPE

# PYTHAGORAS AND TRIGONOMETRY IN 2D AND 3D

Right-angled triangles are everywhere, even inside boxes. Secure Pythagoras and SOH CAH TOA, extend to elevation and bearings, then master lengths and angles in 3D.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

## CASE BRIEFING

### WHAT YOU NEED TO KNOW

**Pythagoras**: in a right-angled triangle, $a^2 + b^2 = c^2$, where $c$ is the hypotenuse (opposite the right angle). Add to find the hypotenuse; subtract to find a shorter side.

**SOH CAH TOA**: $\sin = \frac{\text{opp}}{\text{hyp}}$, $\cos = \frac{\text{adj}}{\text{hyp}}$, $\tan = \frac{\text{opp}}{\text{adj}}$. In **3D**, find a right-angled triangle inside the shape, often using a diagonal of the base first.

1. Find a right-angled triangle and sketch it flat.
2. Label hyp, opp and adj from the angle you're using.
3. Two sides, no angle: Pythagoras. An angle involved: SOH CAH TOA.
4. In 3D, work out the base diagonal first, then use it in a vertical triangle.

**WORKED EXAMPLE** **A cuboid measures 3 cm by 4 cm by 12 cm. Find the length of the longest diagonal.**

### SOLUTION

1 The base is 3 cm by 4 cm. Its diagonal: $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$ cm.

2 The longest diagonal makes a right-angled triangle with the base diagonal (5 cm) and the height (12 cm).

3 Diagonal = $\sqrt{(5^2 + 12^2)} = \sqrt{169}$.

4 = 13 cm. In one step: $\sqrt{(3^2 + 4^2 + 12^2)} = 13$ ✓

**✓ 13 cm**

## KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

**Securing**
Pythagoras and SOH CAH TOA in one triangle.
Secured ☐

**Extending**
Two-step problems, elevation, bearings, and tampered evidence.
Secured ☐

**Mastery**
Lengths and angles inside cuboids and pyramids.
Secured ☐

### SECURING

The method, step by step. Show every line.

**1** A right-angled triangle has shorter sides of 9 cm and 12 cm. Find the hypotenuse. [2]

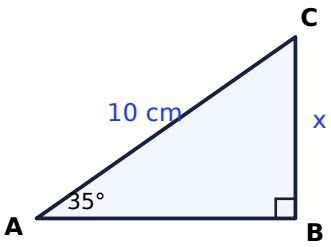
cm _____

**2** A right-angled triangle has a hypotenuse of 13 cm and one side of 5 cm. Find the other side. [2]

cm _____

**FORENSIC TIP** Finding a shorter side means **subtract**. If your answer is longer than the hypotenuse, you added.

**3** Find the length x. Give your answer to 3 significant figures. [2]




Not drawn accurately

cm _____

**4** A right-angled triangle has an opposite side of 7 cm and an adjacent side of 10 cm. Find the angle. [2]

° _____

**5** A 5 m ladder leans against a wall with its foot 1.4 m from the wall. How high up the wall does it reach? [2]

m _____

## EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

**6** An isosceles triangle has sides 10 cm, 10 cm and 12 cm. Find its area. [3]

$cm^2$ _____

**7** A drone hovers 40 m above the ground. An officer stands 65 m from the point directly below it. Find the angle of elevation of the drone. [2]

° _____

**8** A rectangle measures 8 cm by 5 cm. Find the angle between a diagonal and a longer side. [2]

° _____

**9** A suspect walks 6 km north, then 8 km east. Find how far she is from her start, and the bearing she would need to walk back on. [3]

___ km, bearing _____

**10** Rui found the missing side of a right-angled triangle with hypotenuse 10 cm and one side 6 cm. Find his mistake. [2]

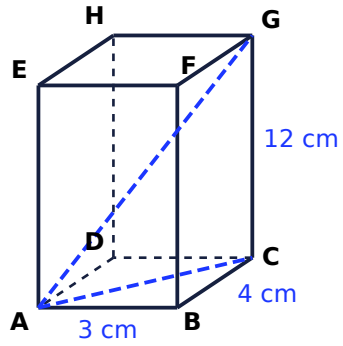
TAMPERED EVIDENCE · FIND THE MISTAKE

$\sqrt{(10^2 + 6^2)} = \sqrt{136} = 11.7$ cm

cm _____

**MASTERY** No hints. Plan before you calculate.

**11** The cuboid ABCDEFGH measures 3 cm by 4 cm by 12 cm. Find the length AG. [3]




cm _____

**12** For the same cuboid, find the angle between AG and the base ABCD. [3]

° _____

**FORENSIC TIP** The angle with the base is in triangle ACG: AC on the base, CG straight up. Sketch that triangle flat.

**13** A square-based pyramid has a base of side 6 cm. The apex is directly above the centre of the base, and each sloping edge is 7 cm. Find the height of the pyramid. [4]

cm _____

**14** For the same pyramid, find the angle between a sloping edge and the base. [2]

° _____

**15** A right-angled triangle has shorter sides x cm and (x + 7) cm, and a hypotenuse of 13 cm. Find x. [4]

x = _____

## KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

**Q1** SECURING

√(81 + 144).

✓ **15 cm**

**Q2** SECURING

√(169 − 25).

✓ **12 cm**

**Q3** SECURING

x = 10 sin 35°.

✓ **5.74 cm**

**Q4** SECURING

$\tan^{-1}(0.7)$.

✓ **35.0°**

**Q5** SECURING

√(25 − 1.96).

✓ **4.8 m**

**Q6** EXTENDING

Height √(100 − 36) = 8. ½ × 12 × 8.

✓ **48 cm²**

**Q7** EXTENDING

$\tan^{-1}(40 \div 65)$.

✓ **31.6°**

**Q8** EXTENDING

$\tan^{-1}(5 \div 8)$.

✓ **32.0°**

**Q9** EXTENDING

The start-to-finish bearing is $\tan^{-1}(8/6)$ = 053°; the way back is 053° + 180°.

✓ **10 km on a bearing of 233°**

**Q10** EXTENDING

He added instead of subtracting: √(100 − 36) = 8.

✓ **8 cm**

**Mastery solutions are in the Lab.** Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-GEO-PYTHTRIG

### Mark your evidence

| Level | Marks | My score | Secured? (every mark) |
|---|---|---|---|
| Securing | 10 | | ☐ |
| Extending | 12 | | ☐ |
| Mastery | 16 | | ☐ |

**R · Reading**
Misread or missed part of the question

**M · Method**
Didn't know how to do it

**A · Accuracy**
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

## CASE CLOSED: PYTHAGORAS AND TRIGONOMETRY IN 2D AND 3D

In this case file you've learned to:

- Use Pythagoras to find any side of a right-angled triangle.
- Choose sin, cos or tan and find sides and angles.
- Solve problems with elevation, bearings and two steps.
- Find lengths and angles inside cuboids and pyramids.
- Form and solve an equation from Pythagoras.

**Because every case comes down to one principle: numbers don't lie.**


YOUR NEXT ASSIGNMENT

### Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab