MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-ALG-QUADEQ
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · ALGEBRA

QUADRATIC EQUATIONS

Three ways to crack a quadratic. Secure factorising, extend to the formula and completing the square, then master disguised quadratics and the discriminant.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

A quadratic equation has an x^2 term. Rearrange to **= 0** first. Then: **factorise** if it factorises, use the **formula** when it doesn't, and **complete the square** when the question asks for surd form or a turning point.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** $b^2 - 4ac$ tells you how many solutions there are: positive, two; zero, one; negative, none.

1. Rearrange so one side is 0.
2. Choose: factorise, formula, or complete the square.
3. Solve each bracket, or substitute carefully into the formula, with brackets round negatives.
4. Give both solutions, rounded as asked, and reject any that don't fit a context.

WORKED EXAMPLE **Solve $2x^2 - 3x - 7 = 0$, giving your answers to 2 decimal places.**

SOLUTION

1 It doesn't factorise, so use the formula with $a = 2$, $b = -3$, $c = -7$.

2 $$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 2 \times (-7)}}{2 \times 2} = \frac{3 \pm \sqrt{65}}{4}$$

3 $\sqrt{65} = 8.0623$, so $x = 11.0623 \div 4$ or $-5.0623 \div 4$.

4 Check by substituting one answer: $2(2.77)^2 - 3(2.77) - 7 \approx 0$ ✓

✓ **$x = 2.77$ or $x = -1.27$**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Solving by factorising, including a ≠ 1.
Secured ☐

Extending
The formula, completing the square, forming a quadratic, and tampered evidence.
Secured ☐

Mastery
Disguised quadratics, the discriminant and context problems.
Secured ☐

SECURING

The method, step by step. Show every line.

1 Solve $x^2 + 7x + 12 = 0$. [3]

x = _____

2 Solve $x^2 - 5x = 0$. [2]

x = _____

FORENSIC TIP Don't divide both sides by x: you lose the solution $x = 0$. Factorise instead.

3 Solve $x^2 - 49 = 0$. [2]

x = _____

4 Solve $x^2 - 2x - 15 = 0$. [3]

x = _____

5 Solve $2x^2 + 7x + 3 = 0$. [3]

x = _____

EXTENDING A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 Solve $3x^2 - 5x - 4 = 0$. Give your answers to 2 decimal places. [3]

x = _____

7 Write $x^2 + 6x - 5$ in the form $(x + a)^2 + b$. [2]

Answer _____

8 Hence solve $x^2 + 6x - 5 = 0$, giving your answers in surd form. [2]

x = _____

9 A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm². Find x. [4]

x = _____

10 Leah solved $x^2 - 3x - 10 = 0$. Find her mistake and give the correct solutions. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$(x - 5)(x + 2) = 0$

$x = -5$ or $x = 2$

x = _____

MASTERY No hints. Plan before you calculate.

11 Solve $\frac{2}{x+1} + \frac{3}{x} = 2$. Give your answers to 2 decimal places. [5]

$x =$ _____

12 Show that $x^2 - 4x + 7 = 0$ has no real solutions. [2]

13 $x^2 + kx + 9 = 0$ has exactly one solution. Find the possible values of k. [3]

$k =$ _____

14 Solve $x^4 - 13x^2 + 36 = 0$. [4]

$x =$ _____

FORENSIC TIP A quadratic in disguise. Substitute $y = x^2$, solve, then take **both** square roots of each answer.

15 A drone's height in metres after t seconds is $h = 20t - 5t^2$. Find when it is 15 m above the ground. [3]

$t =$ _____

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

$(x + 3)(x + 4) = 0$.

✓ **$x = -3$ or $x = -4$**

Q2 SECURING

$x(x - 5) = 0$.

✓ **$x = 0$ or $x = 5$**

Q3 SECURING

Difference of two squares, or $x^2 = 49$.

✓ **$x = 7$ or $x = -7$**

Q4 SECURING

$(x - 5)(x + 3) = 0$.

✓ **$x = 5$ or $x = -3$**

Q5 SECURING

$(2x + 1)(x + 3) = 0$.

✓ **$x = -\frac{1}{2}$ or $x = -3$**

Q6 EXTENDING

$$x = \frac{5 \pm \sqrt{73}}{6}$$

✓ **$x = 2.26$ or $x = -0.59$**

Q7 EXTENDING

$(x + 3)^2 = x^2 + 6x + 9$, so subtract 14.

✓ **$(x + 3)^2 - 14$**

Q8 EXTENDING

$(x + 3)^2 = 14$.

✓ **$x = -3 \pm \sqrt{14}$**

Q9 EXTENDING

$x^2 + 3x - 70 = 0 \rightarrow (x + 10)(x - 7) = 0$. Reject -10: the width would be negative.

✓ **$x = 7$**

Q10 EXTENDING

Her factors are right but the signs of the solutions are flipped: $x - 5 = 0$ gives $x = 5$.

✓ **$x = 5$ or $x = -2$**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-ALG-QUADEQ

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	13		☐
Extending	13		☐
Mastery	17		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: QUADRATIC EQUATIONS

In this case file you've learned to:

- Solve quadratics by factorising, including when a isn't 1.
- Use the quadratic formula and round sensibly.
- Complete the square and give answers in surd form.
- Use the discriminant to count solutions.
- Spot a quadratic in disguise and solve problems in context.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab