MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-ALG-QUADGRAPH
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · GRAPHS

QUADRATIC GRAPHS

Every quadratic graph is a parabola with roots, an intercept and a turning point. Secure reading them, extend to finding them algebraically, then master building and using the equation.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

The graph of $y = ax^2 + bx + c$ is a U shape (a ∩ if a is negative). The **roots** are where it crosses the x-axis: solve $y = 0$. The **y-intercept** is $(0, c)$.

The **turning point** sits on the line of symmetry, halfway between the roots. Completing the square gives it directly: $y = (x - p)^2 + q$ has turning point (p, q).

1. Roots: factorise and solve = 0 (or read where the curve crosses the x-axis).
2. y-intercept: put x = 0.
3. Turning point: halfway between the roots, or complete the square.
4. To solve an equation with a graph, rearrange it so one side is the curve, and draw the other side as a line.

WORKED EXAMPLE **Find the roots, the y-intercept and the turning point of $y = x^2 - 6x + 5$.**

SOLUTION

1 Roots: $(x - 1)(x - 5) = 0$, so $x = 1$ and $x = 5$.

2 y-intercept: when $x = 0$, $y = 5$, so $(0, 5)$.

3 Line of symmetry halfway between the roots: $x = 3$.

4 Turning point: $y = 9 - 18 + 5 = -4$, so $(3, -4)$. Check: $(x - 3)^2 - 4 = x^2 - 6x + 5$ ✓

✓ **Roots 1 and 5, y-intercept (0, 5), turning point (3, −4)**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Tables of values, roots, intercept and turning point from a graph.
Secured ☐

Extending
Roots and turning points by algebra, sketching, symmetry, and tampered evidence.
Secured ☐

Mastery
Finding the equation, solving with a line, and the discriminant.
Secured ☐

SECURING

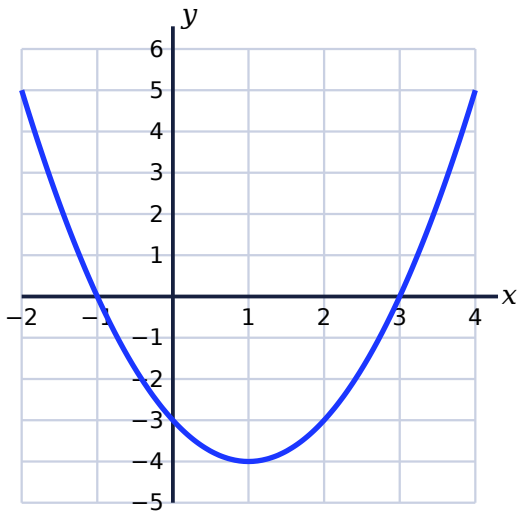
The method, step by step. Show every line.

1 Complete the values of $y = x^2 - 2x - 3$ for $x = -2, -1, 0, 1, 2, 3, 4$. [2]

y values _____

FORENSIC TIP Put negative numbers in brackets when you square them: $(-2)^2 = 4$.

2 The graph of $y = x^2 - 2x - 3$ is drawn below. Write down its roots. [2]



Roots _____

3 Use the graph to write down the coordinates of the turning point. [2]

(___ , ___) _____

4 Write down where the graph of $y = x^2 + 5x + 6$ crosses the y-axis. [1]

(___ , ___) _____

5 Use the graph to solve $x^2 - 2x - 3 = 5$. [2]

x = _____

EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 Find the roots of $y = x^2 - 6x + 8$ by factorising. [2]

Roots _____

7 By completing the square, find the turning point of $y = x^2 - 6x + 8$. [3]

(___ , ___) _____

8 Write down the equation of the line of symmetry of $y = x^2 - 6x + 8$. [1]

Equation _____

9 Sketch $y = x^2 + 2x - 8$. Label the roots, the y-intercept and the turning point with their coordinates. [4]

10 Maya said where $y = (x + 2)(x - 5)$ crosses the x-axis. Find her mistake and give the correct values. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

It crosses at $x = 2$ and $x = -5$

x = _____

MASTERY No hints. Plan before you calculate.

11 The graph of $y = x^2 + bx + c$ has its turning point at $(2, -9)$. Find b and c, and the roots. [4]

b = ___ c = _____

12 A quadratic graph crosses the x-axis at 2 and 3, and the y-axis at $(0, 6)$. Find its equation. [3]

y = _____

13 Use the graph of $y = x^2 - 2x - 3$ and a straight line to solve $x^2 - 3x - 1 = 0$. Write down the equation of your line, then the solutions to 1 decimal place. [3]

Line ___ Solutions _____

FORENSIC TIP Make the left side match the graph you have: add and subtract until it says $x^2 - 2x - 3$. What's left is your line.

14 For which values of k does the graph of $y = x^2 - 4x + k$ not cross the x-axis? [3]

k _____

15 The path of a ball is $y = -0.2x^2 + 2x + 1.5$, where y is its height in metres. Find its greatest height. [3]

m _____

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

Watch $(-2)^2 = 4$, not -4.

✓ **5, 0, −3, −4, −3, 0, 5**

Q2 SECURING

Where the curve crosses the x-axis.

✓ **$x = -1$ and $x = 3$**

Q3 SECURING

✓ **(1, −4)**

Q4 SECURING

✓ **(0, 6)**

Q5 SECURING

Read where the curve is at height 5.

✓ **$x = -2$ and $x = 4$**

Q6 EXTENDING

$(x - 2)(x - 4) = 0$.

✓ **$x = 2$ and $x = 4$**

Q7 EXTENDING

$(x - 3)^2 - 9 + 8 = (x - 3)^2 - 1$.

✓ **(3, −1)**

Q8 EXTENDING

✓ **$x = 3$**

Q9 EXTENDING

$(x + 4)(x - 2)$; $(x + 1)^2 - 9$.

✓ **Roots (−4, 0) and (2, 0); y-intercept (0, −8); turning point (−1, −9)**

Q10 EXTENDING

Each bracket = 0: $x + 2 = 0$ gives $x = -2$.

✓ **$x = -2$ and $x = 5$**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-ALG-QUADGRAPH

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	9		☐
Extending	12		☐
Mastery	16		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: QUADRATIC GRAPHS

In this case file you've learned to:

- Draw and read quadratic graphs from a table of values.
- Find roots by factorising and the y-intercept from c.
- Find the turning point from symmetry or by completing the square.
- Solve equations using the graph and a straight line.
- Build the equation of a quadratic from its features.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab