MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-ALG-SIMEQ
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · ALGEBRA

SIMULTANEOUS EQUATIONS

Two equations, two unknowns, one answer that fits both. Secure the method, extend it to tougher pairs, then master the quadratic cases that decide grade 8 and 9.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

Simultaneous equations are two equations that are both true at the same time. The solution is the pair of values, x and y, that fits **both**. On a graph, it's where the two lines (or the line and the curve) cross.

Elimination works for two linear equations. **Substitution** works when one equation already says y = ..., and always when one equation is a quadratic.

1. Make the coefficients of one letter the same size. Multiply **every** term, including the right-hand side.
2. **Same signs, subtract. Different signs, add.** One letter disappears.
3. Solve for the letter that's left, then substitute back to find the other.
4. Check both values in the equation you **didn't** use. If it works, the case is closed.

WORKED EXAMPLE **A forensic unit orders kit. 3 cameras and 2 lamps cost £430. 1 camera and 2 lamps cost £210. Find the price of a camera and of a lamp.**

SOLUTION

1 Let c = price of a camera, l = price of a lamp: $3c + 2l = 430$ and $c + 2l = 210$

2 The l terms match ($2l$) with the same signs, so subtract: $2c = 220$, so $c = 110$.

3 Substitute into the second equation: $110 + 2l = 210$, so $2l = 100$ and $l = 50$.

4 Check in the first: $3(110) + 2(50) = 330 + 100 = 430$ ✓

✓ **A camera costs £110 and a lamp costs £50.**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Linear pairs: the method, step by step.
Secured ☐

Extending
Scale both equations, contexts, and one piece of tampered evidence.
Secured ☐

Mastery
Quadratic pairs, unknown constants and proof.
Secured ☐

SECURING

Linear pairs. Show your elimination or substitution step.

1 Solve the simultaneous equations [3]

$$x + y = 10$$
$$x - y = 4$$

x = ___ y = _____

2 Solve the simultaneous equations [3]

$$2x + y = 11$$
$$x + y = 7$$

x = ___ y = _____

3 Solve the simultaneous equations [3]

$$3x + 2y = 16$$
$$x - 2y = 0$$

x = ___ y = _____

4 Solve the simultaneous equations [3]

$$5x + 3y = 21$$
$$2x + 3y = 12$$

x = ___ y = _____

5 Two fingerprint kits and one swab pack cost £31. One fingerprint kit and one swab pack cost £19.

Work out the cost of a fingerprint kit and the cost of a swab pack. [3]

Kit £ ___ Swabs £ _____

FORENSIC TIP Write your two equations first, with letters you define (k = cost of a kit). In the exam, the first method mark is for the equations.

EXTENDING

Scale one or both equations first. Watch every sign.

6 Solve the simultaneous equations [4]

$$3x + 4y = 5$$
$$2x - 3y = 9$$

x = ___ y = _____

7 Solve the simultaneous equations [4]

$$4x - 5y = 7$$
$$3x + 2y = 11$$

x = ___ y = _____

8 Passes for a crime-lab conference: 3 analyst passes and 5 visitor passes cost £61. 2 analyst passes and 3 visitor passes cost £39.50. Work out the cost of each pass. [4]

Analyst £ ___ Visitor £ _____

9 A suspect's route follows the line $y = 2x - 3$. A patrol car follows $3x + 2y = 15$. Find the coordinates of the point where their paths cross. [3]

(___ , ___) _____

10 Priya solved these equations. Find her mistake, then solve them correctly. [3]

$2x + 3y = 12$

$2x - y = 4$

TAMPERED EVIDENCE · FIND THE MISTAKE

Subtract: $2y = 8$, so $y = 4$

$2x + 12 = 12$, so $x = 0$

x = ___ y = _____

FORENSIC TIP Subtracting a negative is the most common slip in elimination. Write the subtraction in a column, term by term.

MASTERY

Quadratics, unknown constants and proof. Plan before you calculate.

11 A surveillance zone is the circle $x^2 + y^2 = 25$. A drone flies along the line $y = x + 1$. [5]

Find the two points where the drone crosses the edge of the zone.

Points _____

FORENSIC TIP Expand $(x + 1)^2$ as $x^2 + 2x + 1$, not $x^2 + 1$. Then give **both** points, each x matched with its own y.

12 Find the coordinates of the points where the curve $y = x^2 - 3x + 2$ meets the line $y = 2x - 4$. [4]

Points _____

13 The line $y = x + k$ touches the curve $y = x^2 - 3x + 6$ at exactly one point. [4]

Find the value of k.

k = _____

14 Two codes add up to 4. The sum of their squares is 58. [4]

Find the two codes.

The codes are _____

15 Show that these simultaneous equations have no solution. Explain what this means for their graphs. [2]

$$2x + 3y = 6$$
$$4x + 6y = 15$$

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

Different signs, add: $2x = 14$.

✓ **$x = 7$, $y = 3$**

Q2 SECURING

Same signs, subtract: $x = 4$.

✓ **$x = 4$, $y = 3$**

Q3 SECURING

Different signs, add: $4x = 16$.

✓ **$x = 4$, $y = 2$**

Q4 SECURING

Same signs, subtract: $3x = 9$.

✓ **$x = 3$, $y = 2$**

Q5 SECURING

$2k + s = 31$, $k + s = 19$. Subtract: $k = 12$.

✓ **Kit £12, swab pack £7**

Q6 EXTENDING

×3 and ×4: $9x + 12y = 15$, $8x - 12y = 36$. Add: $17x = 51$.

✓ **$x = 3$, $y = -1$**

Q7 EXTENDING

×2 and ×5: $8x - 10y = 14$, $15x + 10y = 55$. Add: $23x = 69$.

✓ **$x = 3$, $y = 1$**

Q8 EXTENDING

×2 and ×3: $6a + 10v = 122$, $6a + 9v = 118.50$. Subtract: $v = 3.50$.

✓ **Analyst £14.50, visitor £3.50**

Q9 EXTENDING

$3x + 2(2x - 3) = 15$, so $7x = 21$.

✓ **(3, 3)**

Q10 EXTENDING

$3y - (-y) = 4y$, not $2y$. So $4y = 8$, $y = 2$, then $2x + 6 = 12$.

✓ **$x = 3$, $y = 2$**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-ALG-SIMEQ

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	15		☐
Extending	18		☐
Mastery	19		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: SIMULTANEOUS EQUATIONS

In this case file you've learned to:

- Solve linear pairs by elimination, scaling one or both equations first.
- Use substitution when one equation is a quadratic, and give every pair of answers.
- Turn a word problem into two equations with letters you define.
- Spot the sign slip in someone else's working before it costs you marks.
- Use the discriminant to decide when a line only touches a curve.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab