MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-GEO-SINECOS
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · SHAPE

SINE AND COSINE RULE

Trigonometry for triangles without a right angle. Secure choosing the right rule, extend to areas and contexts, then master the ambiguous case and algebra.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

Label the sides with lower-case letters opposite their angles: side a is opposite angle A.

Sine rule (a pair of opposite side and angle, plus one more): $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. **Cosine rule** (two sides and the angle between, or all three sides): $a^2 = b^2 + c^2 - 2bc \cos A$. **Area** = $\frac{1}{2}ab \sin C$.

1. Label the triangle: angles A, B, C; sides a, b, c opposite them.
2. Have you got a side and its opposite angle? Sine rule. Otherwise, cosine rule.
3. Substitute, then rearrange before you press any buttons.
4. Round only at the end, and check the biggest side is opposite the biggest angle.

WORKED EXAMPLE **In triangle PQR, PQ = 9 cm, PR = 7 cm and angle P = 50°. Find QR.**

SOLUTION

1 Two sides and the angle between them, with no opposite pair: cosine rule.

2 $QR^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos 50°$.

3 $= 81 + 49 - 126 \times 0.6428 = 49.01$.

4 $QR = \sqrt{49.01} = 7.00$ cm. Check: it's opposite 50°, between the other two sides ✓

✓ **QR = 7.00 cm (3 s.f.)**

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Sine rule, cosine rule and ½ab sin C, one step each.
Secured ☐

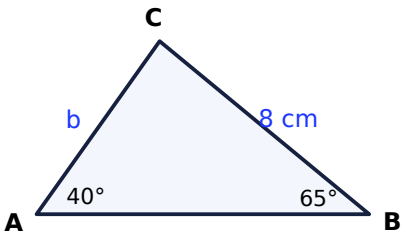
Extending
Angles, quadrilaterals, contexts, and tampered evidence.
Secured ☐

Mastery
Two possible triangles, bearings, and forming equations.
Secured ☐

SECURING

The method, step by step. Show every line.

1 Find the length *b*. Give your answer to 3 significant figures. [3]



Not drawn accurately

cm _____

2 In triangle ABC, b = 7 cm, c = 9 cm and angle A = 50°. Find a. [3]

cm _____

3 Find the area of a triangle with sides 6 cm and 9 cm and an angle of 40° between them. [2]

cm^2 _____

4 A triangle has sides 7 cm, 8 cm and 9 cm. Find the largest angle. [3]

° _____

FORENSIC TIP The largest angle is opposite the longest side. Rearrange first: $\cos C = (a^2 + b^2 - c^2) \div 2ab$.

5 In triangle ABC, a = 10 cm, angle A = 70° and b = 8 cm. Find angle B. [3]

° _____

EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 Two observers are 200 m apart on a straight road. They see a suspect's drone at angles of 50° and 60° from the road. How far is the drone from the first observer? [3]

m _____

FORENSIC TIP Sketch it first. The side you want is opposite the angle at the **second** observer.

7 A triangle has sides 5 cm, 6 cm and 7 cm. Find its area. [4]

cm^2 _____

8 A parallelogram has sides of 8 cm and 5 cm, with angles of 60° and 120°. Find the length of its longer diagonal. [3]

cm _____

9 In triangle XYZ, XY = 12 cm, angle X = 35° and angle Y = 80°. Find XZ. [3]

cm _____

10 Jas used the cosine rule with b = 7, c = 9 and A = 50°. Find her mistake. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$a^2 = (49 + 81 - 126)\cos 50° = 2.57$

$a = 1.60$ cm

Correct a _____

MASTERY

No hints. Plan before you calculate.

11 In triangle ABC, a = 7 cm, b = 9 cm and angle A = 40°. Find the two possible sizes of angle B. [3]

° _____

FORENSIC TIP When the side opposite the known angle is the **shorter** one, two triangles can fit. Check whether 180° minus your angle also works.

12 A triangle has an area of 30 cm². Two of its sides are 8 cm and 10 cm. Find the acute angle between them. [3]

° _____

13 A patrol boat sails 12 km on a bearing of 060°, then 8 km on a bearing of 130°. How far is it from its start? [4]

km _____

14 A triangle has sides x cm, (x + 1) cm and (x + 2) cm. The largest angle is 120°. Find x. [4]

x = _____

15 Show that the area of an equilateral triangle with side s is $(\sqrt{3}/4)s^2$. [2]

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

b = 8 sin 65° ÷ sin 40°.

✓ **11.3 cm**

Q2 SECURING

$a^2 = 49 + 81 - 126 \cos 50°$.

✓ **7.00 cm**

Q3 SECURING

½ × 6 × 9 × sin 40°.

✓ **17.4 cm²**

Q4 SECURING

cos C = (49 + 64 − 81) ÷ 112.

✓ **73.4°**

Q5 SECURING

sin B = 8 sin 70° ÷ 10.

✓ **48.7°**

Q6 EXTENDING

The third angle is 70°. 200 sin 60° ÷ sin 70°.

✓ **184 m**

Q7 EXTENDING

Find one angle by the cosine rule (e.g. 78.5° between 5 and 6), then ½ × 5 × 6 × sin 78.5°.

✓ **14.7 cm²**

Q8 EXTENDING

$d^2 = 64 + 25 - 80 \cos 120°$.

✓ **11.4 cm**

Q9 EXTENDING

Angle Z = 65°. 12 sin 80° ÷ sin 65°.

✓ **13.0 cm**

Q10 EXTENDING

She multiplied the whole of $b^2 + c^2 - 2bc$ by cos A. Only 2bc is multiplied by cos A.

✓ **a = 7.00 cm**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-GEO-SINECOS

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	14		☐
Extending	15		☐
Mastery	16		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: SINE AND COSINE RULE

In this case file you've learned to:

- Label any triangle and choose between the sine and cosine rules.
- Find missing sides and angles in triangles without a right angle.
- Find the area of any triangle with ½ab sin C.
- Spot the ambiguous case and give both angles.
- Use the rules with bearings and with algebra.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab