MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-NUM-SURDS
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · NUMBER

SURDS

Exact answers, no rounding. Secure simplifying and multiplying, extend to brackets and rationalising, then master surd algebra in shapes and equations.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

A surd is a root that isn't a whole number, like $\sqrt{2}$. Keep it as a surd and the answer stays **exact**.

Key facts: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, and $\sqrt{a} \times \sqrt{a} = a$. To **simplify**, find the biggest square number that's a factor. To **rationalise** a denominator, multiply top and bottom by the surd, or by the **conjugate** if the denominator is a bracket: for $3 - \sqrt{2}$, multiply by $3 + \sqrt{2}$.

1. Look for the largest square factor: 4, 9, 16, 25, 36, 49...
2. Only like surds add: $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$, but $\sqrt{2} + \sqrt{3}$ can't be simplified.
3. Expand brackets term by term, just like algebra.
4. Rationalise: multiply top and bottom by the same thing, so the value doesn't change.

WORKED EXAMPLE Rationalise the denominator of $\dfrac{10}{3 - \sqrt{2}}$.

SOLUTION

1 The denominator is a bracket, so multiply top and bottom by the conjugate, $3 + \sqrt{2}$.

2 Top: $10(3 + \sqrt{2}) = 30 + 10\sqrt{2}$.

3 Bottom: $(3 - \sqrt{2})(3 + \sqrt{2}) = 9 - 2 = 7$. The surds cancel.

4 So the answer is $(30 + 10\sqrt{2}) \div 7$.

✓ $\dfrac{30 + 10\sqrt{2}}{7}$

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Simplify, add, multiply, and rationalise a simple denominator.
Secured ☐

Extending
Expanding brackets, conjugates, and tampered evidence.
Secured ☐

Mastery
Rationalising harder fractions, surds in shapes and equations.
Secured ☐

SECURING

The method, step by step. Show every line.

1 Simplify $\sqrt{50}$. [2]

Answer _____

2 Simplify $\sqrt{12} + \sqrt{27}$. [2]

Answer _____

3 Simplify $\sqrt{3} \times \sqrt{12}$. [2]

Answer _____

4 Expand and simplify $\sqrt{5}(2 + \sqrt{5})$. [2]

Answer _____

FORENSIC TIP $\sqrt{5} \times \sqrt{5} = 5$. A common slip is writing 25, or leaving it as $\sqrt{25}$.

5 Rationalise the denominator of $\frac{6}{\sqrt{3}}$. [2]

Answer _____

EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 Expand and simplify $(3 + \sqrt{2})(3 - \sqrt{2})$. [2]

Answer _____

7 Expand and simplify $(2 + \sqrt{3})^2$. [2]

Answer _____

FORENSIC TIP Write the bracket out twice. $(2 + \sqrt{3})^2$ is not $4 + 3$.

8 Rationalise the denominator of $\dfrac{4}{\sqrt{5} - 1}$. Simplify your answer. [3]

Answer _____

9 A rectangle measures $(3 + \sqrt{5})$ cm by $(3 - \sqrt{5})$ cm. Find its area. [2]

cm² _____

10 Ali expanded $(\sqrt{3} + 1)^2$. Find his mistake and give the correct answer. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$(\sqrt{3} + 1)^2 = 3 + 1 = 4$

Answer _____

MASTERY

No hints. Plan before you calculate.

11 Rationalise the denominator of $\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}}$. Give your answer in the form $a + b\sqrt{3}$. [3]

Answer _____

12 A right-angled triangle has shorter sides $(2 + \sqrt{3})$ cm and $(2 - \sqrt{3})$ cm. Find the exact length of the hypotenuse. [3]

cm _____

13 Solve $x\sqrt{2} + 3 = x\sqrt{8} - 5$. Give your answer in the form $k\sqrt{2}$. [4]

x = _____

FORENSIC TIP Simplify √8 first. Then you have like surds you can collect.

14 Show that $\dfrac{\sqrt{12} + \sqrt{75}}{\sqrt{3}}$ is a whole number, and find it. [2]

Answer _____

15 Show that $(\sqrt{a} + \sqrt{b})^2 - (\sqrt{a} - \sqrt{b})^2 = 4\sqrt{ab}$. [2]

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

√(25 × 2).

✓ **5√2**

Q2 SECURING

2√3 + 3√3.

✓ **5√3**

Q3 SECURING

√36.

✓ **6**

Q4 SECURING

√5 × √5 = 5.

✓ **2√5 + 5**

Q5 SECURING

Multiply top and bottom by √3: 6√3 ÷ 3.

✓ **2√3**

Q6 EXTENDING

9 − 2: the surd terms cancel.

✓ **7**

Q7 EXTENDING

4 + 4√3 + 3.

✓ **7 + 4√3**

Q8 EXTENDING

× (√5 + 1): 4(√5 + 1) ÷ (5 − 1).

✓ **√5 + 1**

Q9 EXTENDING

9 − 5.

✓ **4 cm²**

Q10 EXTENDING

He squared each term but missed the middle terms: 3 + 2√3 + 1.

✓ **4 + 2√3**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-NUM-SURDS

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	10		☐
Extending	11		☐
Mastery	14		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: SURDS

In this case file you've learned to:

- Simplify a surd using its largest square factor.
- Add, subtract and multiply surds.
- Expand brackets containing surds, using the conjugate to clear them.
- Rationalise any denominator, including a bracket.
- Keep answers exact in shapes and equations.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab