MATHS FORENSICS
CASE FILE WORKSHEET · GCSE

MF-CF-PROB-VENN
GCSE HIGHER · ABOUT 50 MINUTES

CASE FILE SERIES · DATA AND PROBABILITY

VENN DIAGRAMS

Overlaps are where the marks hide. Secure set notation, extend to surveys and conditional probability, then master Venn diagrams with algebra and three sets.

SECURING · GRADES 4–5 | EXTENDING · GRADES 6–7 | MASTERY · GRADES 8–9

Name _____ Class _____ Date _____

CASE BRIEFING

WHAT YOU NEED TO KNOW

ξ is everything. **A ∩ B** (intersection) is in both. **A ∪ B** (union) is in either or both. **A′** is everything not in A.

Fill a Venn diagram from the **middle outwards**: the overlap first, then 'only A' and 'only B', then outside. For **given that** questions, the total becomes only the part you're told about.

1. Put the overlap in first.
2. Subtract to find 'only A' and 'only B'. Don't count the overlap twice.
3. Check every region adds up to the total in ξ.
4. For P(B given A), divide by the number in A, not the total.

WORKED EXAMPLE **40 officers were asked if they have a dog (D) or a cat (C). 18 have a dog, 15 a cat, and 6 have both. Find the probability that an officer chosen at random has neither.**

SOLUTION

1 Both: 6 in the overlap.

2 Dog only: 18 − 6 = 12. Cat only: 15 − 6 = 9.

3 Inside the circles: 12 + 6 + 9 = 27. Outside: 40 − 27 = 13.

4 P(neither) = $\frac{13}{40}$. Check: 12 + 6 + 9 + 13 = 40 ✓

✓ $\mathbf{\frac{13}{40}}$

KNOWLEDGE CHECK

Use your investigative skills to solve each case. Show full working: the method marks are where most students lose out.

Securing
Set notation: ∩, ∪, complement, and probabilities from a diagram.
Secured ☐

Extending
Surveys with an overlap, conditional probability, and tampered evidence.
Secured ☐

Mastery
Algebra in Venn diagrams, three sets, and independence.
Secured ☐

SECURING

The method, step by step. Show every line.

1 ξ = {1, 2, 3, ..., 12}. A = multiples of 3. B = even numbers. List the members of A ∩ B. [2]

A ∩ B = _____

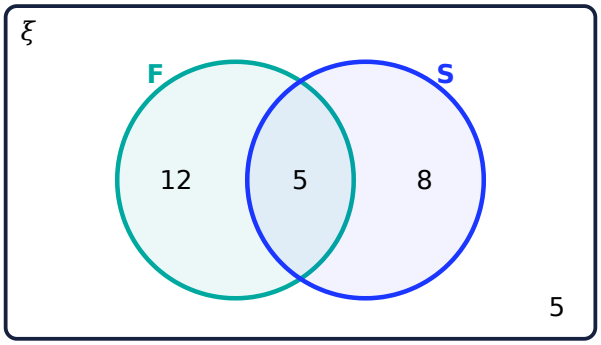
2 Using the same sets, list the members of A ∪ B. [2]

A ∪ B = _____

3 Using the same sets, a number is chosen at random from ξ. Find P((A ∪ B)′). [2]

Answer _____

4 The Venn diagram shows 30 trainees who study French (F) and Spanish (S). Find the probability a trainee chosen at random studies French. [2]



Answer _____

FORENSIC TIP 'Studies French' includes the overlap. It's 12 + 5, not 12.

5 Using the Venn diagram, how many trainees are in S′? [1]

Answer _____

EXTENDING

A step is hidden, or there's a context to unpick. One piece of tampered evidence.

6 50 people were asked about pets. 28 own a dog, 20 own a cat and 9 own both. How many own neither? [3]

Answer _____

7 Using the same survey, find the probability that a person chosen at random owns a cat but not a dog. [2]

Answer _____

8 Using the same survey, a dog owner is chosen at random. Find the probability they also own a cat. [2]

Answer _____

9 $\xi = \{1, 2, \ldots, 12\}$, A = multiples of 3, B = even numbers. List $A' \cap B$. [2]

$A' \cap B$ = _____

10 Nia worked out how many of the 50 people own a pet. Find her mistake. [2]

TAMPERED EVIDENCE · FIND THE MISTAKE

$28 + 20 = 48$ people own a pet

Answer _____

MASTERY No hints. Plan before you calculate.

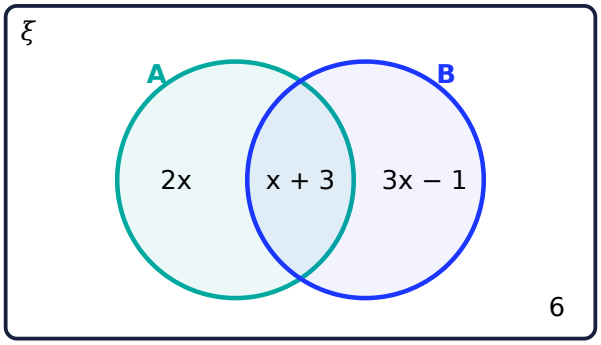
11 40 people were surveyed. 25 are in group A, 22 in group B, and 5 are in neither. How many are in both A and B? [3]

Answer _____

12 Using the same survey, a person in A is chosen at random. Find the probability they are also in B. [2]

Answer _____

13 The Venn diagram shows 50 detectives. Find x, then the probability that a detective chosen at random is in B. [4]



x = ____ P(B) = _____

14 ξ = {1, 2, …, 20}. P = prime numbers, E = even numbers, S = square numbers. How many numbers are in exactly two of the sets? [3]

Answer _____

15 $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$. Are A and B independent? Give a reason. [2]

Yes / No _____

KNOWLEDGE CHECK: SOLUTIONS

Check your method as well as your answer. In the exam, an answer with no working usually earns the final mark at most.

Q1 SECURING

✓ **{6, 12}**

Q2 SECURING

✓ **{2, 3, 4, 6, 8, 9, 10, 12}**

Q3 SECURING

{1, 5, 7, 11}: 4 out of 12.

✓ **1/3**

Q4 SECURING

12 + 5.

✓ **17/30**

Q5 SECURING

12 + 5 outside S.

✓ **17**

Q6 EXTENDING

19 dog only + 9 + 11 cat only = 39. 50 − 39.

✓ **11**

Q7 EXTENDING

✓ **11/50**

Q8 EXTENDING

Given a dog owner: out of 28, not 50.

✓ **9/28**

Q9 EXTENDING

Even, but not a multiple of 3.

✓ **{2, 4, 8, 10}**

Q10 EXTENDING

She counted the 9 who own both twice: 28 + 20 − 9.

✓ **39**

Mastery solutions are in the Lab. Grades 8 and 9 are won on method, so a bare answer won't tell you much. Photograph or write your Mastery working in **Forensic Marking** and get it marked mark by mark, with the full solution and the mistake behind every lost mark. Your first marked question is free.

lab.mathsforensics.com/written/?sheet=CF-PROB-VENN

Mark your evidence

Level	Marks	My score	Secured? (every mark)
Securing	9		☐
Extending	11		☐
Mastery	14		☐

R · Reading
Misread or missed part of the question

M · Method
Didn't know how to do it

A · Accuracy
Knew how, slipped in the working

Write R, M or A next to every mark you lost. Mostly R or A? The fix is technique, not more practice of the topic.

CASE CLOSED: VENN DIAGRAMS

In this case file you've learned to:

- Read and use ∩, ∪ and complement.
- Fill a Venn diagram from the overlap outwards.
- Find probabilities, including 'given that'.
- Use algebra to find missing values in a Venn diagram.
- Test whether two events are independent.

Because every case comes down to one principle: numbers don't lie.

YOUR NEXT ASSIGNMENT

Practise every GCSE topic in the Lab

New numbers every time, feedback that names the mistake behind each wrong answer, and handwritten working marked like an examiner. Start free.

https://www.mathsforensics.com/the-lab